

If it does not kill them, it makes them stronger: collisional evolution of star clusters with tidal shocks

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ABSTRACT

The radii of young ($\lesssim 100$ Myr) star clusters correlate only weakly with their masses. This shallow relation has been used to argue that impulsive tidal perturbations, or ‘shocks’, by passing giant molecular clouds (GMCs) preferentially disrupt low-mass clusters. We show that this mass-radius relation is in fact the result of the combined effect of two-body relaxation and repeated tidal shocks. Clusters in a broad range of environments including those like the solar neighbourhood evolve towards a typical radius of a few parsecs, as observed, independent of the initial radius. This equilibrium mass-radius relation is the result of a competition between expansion by relaxation and shrinking due to shocks. Interactions with GMCs are more disruptive for low-mass clusters, which helps to evolve the globular cluster mass function (GCMF). However, the properties of the interstellar medium in high-redshift galaxies required to establish a universal GCMF shape are more extreme than previously derived, challenging the idea that all GCs formed with the same power-law mass function.

Key words: stars: kinematics and dynamics – ISM: structure – globular clusters: general – open clusters and associations: general –

1 INTRODUCTION

The open clusters in the Milky Way (e.g. Kharchenko et al. 2005; van den Bergh 2006) and young clusters in external spirals (e.g. Zepf et al. 1999; Scheepmaker et al. 2007) have radii of a few pc, almost independent of cluster mass M . Larsen (2004) finds for clusters with ages $\lesssim 100$ Myr and $10^3 \lesssim M/M_\odot \lesssim 10^5$ that the average effective radius r_{eff} , defined as the radius containing half of the light in projection, scales as $r_{\text{eff}} \approx 2.8 \text{ pc } (M/10^4 M_\odot)^{0.1}$. This is strikingly different from the mass-radius relation (MRR) of star forming clumps, from which star clusters presumably form and for which the radius depends strongly on M (Larson 1981). For example, Urquhart et al. (2014) find that the radius of star forming clumps scales with mass as $3.8 \text{ pc } (M/10^4 M_\odot)^{0.6}$. It is not clear whether this difference in MRR of molecular clumps and star clusters originates from the star formation process that alters the relation, or whether it results from subsequent evolutionary effects.

The near constant radius of star clusters has important consequences for their survivability. Interactions with giant molecular clouds (GMCs) disrupt star clusters (Spitzer 1958), and this mechanism has been invoked as an explanation for the dearth of old open clusters in the Milky Way disc (Wielen 1985; Terlevich 1987). Spitzer (1958) shows that the corresponding disruption time-scale is proportional to the cluster density. Fall et al. (2009) argue that clusters form with similar densities, such that GMC encounters disrupt clusters independently of their masses. However, the weak dependence of r_{eff} on M of young clusters implies that low-mass clus-

ters are less dense, and therefore more vulnerable to tidal shocks (Gieles et al. 2006, hereafter G06).

Clusters form in regions with high gas densities and after formation they drift away from these regions, and cloud interactions are therefore more important in the early evolution than estimated from their current environment (Elmegreen & Hunter 2010). Elmegreen (2010) uses this, and the observed MRR, to suggest that young globular clusters in the early Universe with masses of up to $10^5 M_\odot$ were more vulnerable to disruption by gas clouds than more massive GCs. Elmegreen proposes that this early disruption mechanism can evolve a -2 power-law cluster mass distribution, as is observed for young massive clusters (YMCs; Zhang & Fall 1999; Portegies Zwart, McMillan & Gieles 2010), into a universally peaked globular cluster mass function (GCMF; see also Kruijsen 2015).

However, tidal interactions affect not only the clusters’ masses, but also their radii, such that the MRR evolves. The MRR is also affected by internal two-body relaxation, which causes clusters to expand until the galactic tidal field stops the expansion. Here we study cluster evolution as the result of both tidal shocks and two-body relaxation by combining prescriptions for the change in the total cluster energy due to both processes. The total energy of a self-gravitating stellar system in virial equilibrium depends on M and the half-mass radius r_h as $E = -\alpha GM^2/r_h$. Here G is the gravitational constant and α is a form-factor that depends on the density profile of the cluster. Both tidal shocks and two-body relaxation affect the density profile because of redistribution of energy (e.g.

Spitzer 1987; Gnedin, Lee & Ostriker 1999, respectively), but for a wide range of cluster models $\alpha = 0.2$ to within 20% (e.g. Spitzer 1987), hence we fix $\alpha = 0.2$ from hereon. We describe the evolution of the cluster in terms of M and the average density within r_h , ρ_h , so we express E in these quantities: $E \propto -GM^{5/3}\rho_h^{1/3}$. The fractional change in E then relates to variations in M and ρ_h as

$$\frac{dE}{E} = \frac{5}{3} \frac{dM}{M} + \frac{1}{3} \frac{d\rho_h}{\rho_h}. \quad (1)$$

We solve for the evolution of all three variable in two steps: (1) establish how the evolution of M depends on the evolution of E (independent of time) to find an expression for $\rho_h(E)$; (2) find the evolution of E on the appropriate time-scales.

In Sections 2 and 3 we derive the relations for tidal shocks and relaxation, respectively. In Section 4 we combine the two effects and derive an equilibrium MRR and the evolution of all parameter in time. Our conclusions are presented in Section 5.

2 TIDAL SHOCKS

2.1 Density evolution

Here we derive the response of a self-gravitating system to a single tidal perturbation to relate E and M . We assume that the duration of the tidal perturbation is much shorter than the crossing time of stars in the cluster (i.e. the impulsive regime) such that the effect of adiabatic damping (Spitzer 1987; Weinberg 1994; Kundic & Ostriker 1995) can be ignored and the term ‘shock’ applies. This assumption is justified because typical relative velocities during encounters with GMCs are much higher ($\gtrsim 10 \text{ km s}^{-1}$) than the velocities of stars in the outer parts of clusters ($\lesssim 1 \text{ km s}^{-1}$).

The energy gain due to a shock can be expressed in the properties of the cluster, the GMC and the encounter (Spitzer 1958; Binney & Tremaine 1987). These analytic results show good agreement with results for numerical experiments (Gnedin & Ostriker 1999, G06). G06 also provide an expression for the mass loss resulting from a single encounter. However, neither the theory, nor the numerical work, provide a description of what the energy of the remaining bound stars is, which is needed to understand the subsequent response of the cluster density (see equation 1).

To proceed, we introduce a parameter f to relate E and M

$$\frac{dM}{M} = f \frac{dE_{\text{sh}}}{E}. \quad (2)$$

We assume that all stars are bound to the cluster before a shock is applied, such that mass loss always results in an increase of E (i.e. $f > 0$). Substituting equation (2) in equation (1) we find an expression for the relation between ρ_h and E

$$\frac{d\rho_h}{\rho_h} = (3 - 5f) \frac{dE_{\text{sh}}}{E}. \quad (3)$$

Note that combined with equation (2) it is straightforward to express the evolution of ρ_h in terms of M . For $f = 3/5$ the density remains constant and for $f > 3/5$ the cluster density goes up. Similarly, for $f = 1/2$ the cluster evolves with a constant r_h .

To find an estimate for f^1 , we consider the effect of an individual tidal shock. If a tidal force works on a cluster for some time, the

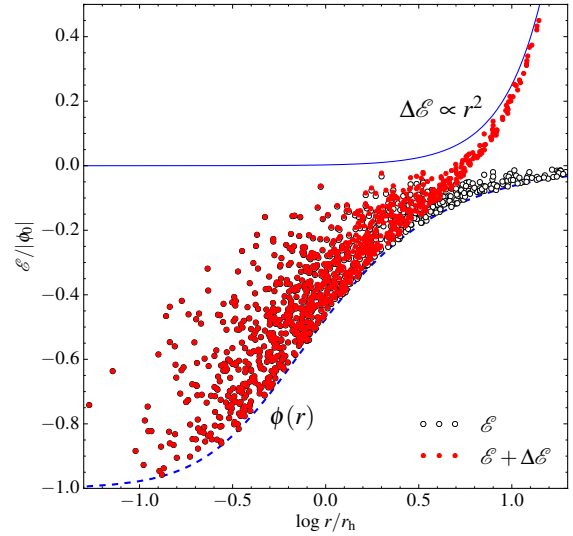


Figure 1. Specific energy (\mathcal{E}) of 10^3 stars, normalized to the central potential ϕ_0 , as a function of distance to the cluster centre in terms of r_h , for an isochrone model (Hénon 1959; Hénon 1960, open circles). The dashed line indicates the specific potential, ϕ . The filled (red) circles represent the energy after a tidal shock, i.e. an additional energy $\Delta\mathcal{E} \propto r^2$. The stars that are unbound after the perturbation, had an energy close to 0 before the encounter. As a result, the average (specific) energy of remaining stars is lower after the shock, and the cluster shrinks.

velocity increase of a star, Δv , is proportional to its distance from the cluster centre r . The increase in the specific energy of the stars \mathcal{E} is then $\Delta\mathcal{E} \propto r^2$ (Spitzer 1958), where the constant of proportionality depends on the strength of the shock. Note that we ignore the cross term $v\Delta v$, which is small compared to $(\Delta v)^2$ for escapers, and we refer to Gnedin & Ostriker (1999) for a discussion.

We add $\Delta\mathcal{E}$ to \mathcal{E} (see Fig. 1) of stars in self-consistent, isotropic, equilibrium models and then find the total M and E of the stars that remain bound (i.e. those for which $\mathcal{E} + \Delta\mathcal{E} \leq 0$). In Fig. 2 we show the results for different shock strengths, for the isochrone model (Hénon 1959; Hénon 1960), the Jaffe model (Jaffe 1983) and the Plummer model (Plummer 1911), all truncated at $100r_h$. Elson, Fall & Freeman (1987) and Mackey & Gilmore (2003) find in a sample of young clusters in the Large Magellanic Cloud that most clusters have luminosity profiles with logarithmic slopes between -2.5 and -3 in the outer regions, corresponding to -3.5 and -4 after de-projecting under the assumption of spherical symmetry. The density profiles of the isochrone and Jaffe models have slopes of -4 at large radii, hence the results for these models are more applicable than that of the Plummer model which has a steeper density profile (r^{-5}). The value of f depends on C and hence on M/M_0 . G06 show that most of the energy gain is due to encounters with a relative velocity comparable to the dispersion of the relative velocity distribution, and with an impact parameter similar to the radius of the GMC (see their fig. 11). With the results of Spitzer (1958) and G06 we find that for such encounters clusters lose a few percent of their mass, or less. Based on this we adopt $f = 3$ from hereon (see

¹ G06 introduced a parameter f that relates energy gain ΔE to mass loss ΔM as the result of a single encounter between a cluster and a GMC as $\Delta M/M_0 = f\Delta E/E_0$, where M_0 and E_0 are the mass and energy of the cluster before the encounter, and they find $f \approx 0.2$. This result applies to the

energy gain of all the stars, including the unbound stars. Stars escape with positive energies, so this result does not give us the required $\Delta E = E_1 - E_0$, where E_1 is the energy of the remaining bound stars.

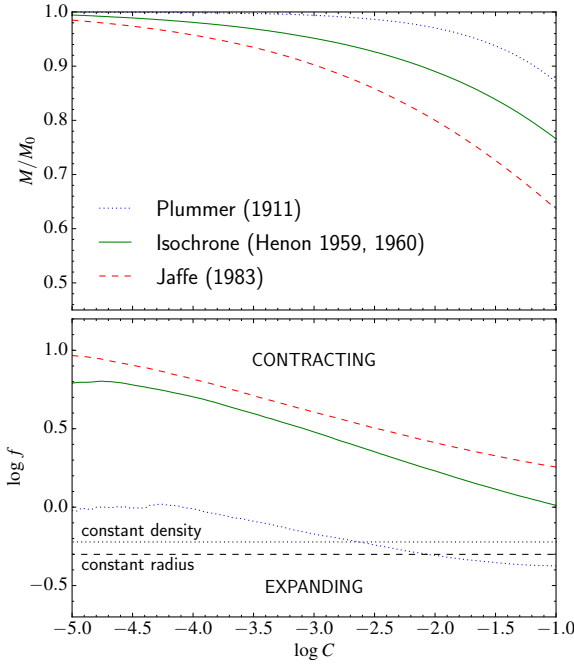


Figure 2. Remaining bound mass fraction (top) and logarithmic slope f of the relation between mass and energy (equation 2) after a single tidal shocks $\Delta E / \langle \phi \rangle = C(r/r_h)^2$, where $\langle \phi \rangle$ is the average specific potential of stars. Results are computed for three different equilibrium models. For the models with density fall-off $\rho \propto r^{-4}$ in the outer parts (Jaffe and isochrone models) the values of f are similar, and larger than the critical value for f to keep ρ_h constant ($f = 3/5$) or r_h constant ($f = 1/2$). This means that clusters contract to a higher density as the result of a tidal shocks.

Fig. 2). In Section 4.2 we show that the results are insensitive to the exact value of f .

We note that the effect of shocks is strongly self-limiting: if the mass reduces by a factor of $q < 1$, the density increases by a factor of q^{-4} (for $f = 3$), making the cluster more susceptible against the next shock. Gnedin et al. (1999) discuss this self-limiting nature of tidal shocks in the context of disc crossings of globular clusters. Most studies on GMC interactions implicitly assume that shocks do not affect the density (i.e. $f = 3/5$, Fall et al. 2009), or only mildly affect the radius (i.e. $f \simeq 1/2$, G06; Elmegreen 2010; Kruijssen 2015). In what follows we show that it is important to include the self-limiting nature of tidal shocks to understand cluster evolution.

2.2 time-scale

We introduce a time-scale τ_{sh} for the change in energy as the result of repeated shocks

$$\frac{dE_{sh}}{E} = -\frac{dt}{\tau_{sh}}, \quad (4)$$

where $\tau_{sh} \propto \rho_h$ (e.g. Spitzer 1958; Ostriker, Spitzer & Chevalier 1972), such that we can write

$$\tau_{sh} = \gamma_{GMC} \frac{\rho_h}{10^2 M_\odot \text{pc}^{-3}}. \quad (5)$$

Here γ_{GMC} is a constant that depends on the dispersion of the relative velocities between GMCs and the cluster (σ_{rel}), the surface

density of individual GMCs (Σ_{GMC}) and the average mass density of clumpy gaseous structures in the interstellar medium (ISM), ρ_{ISM} , as (Spitzer 1958, G06)

$$\gamma_{GMC} \simeq 6.5 \text{ Gyr} \frac{\sigma_{rel}}{10 \text{ km s}^{-1}} \frac{10 M_\odot \text{pc}^{-5}}{\Sigma_{GMC} \rho_{ISM}}. \quad (6)$$

There are several constants that have to be chosen to arrive at the constant of proportionality in equation (6) and we used the values adopted in Section 5.2 of G06 appropriate for King (1966) models with dimensionless central potential $W_0 = 7$. For the Galactic disc ($\sigma_{rel} \simeq 10 \text{ km s}^{-1}$, $\Sigma_{GMC} \simeq 170 M_\odot \text{pc}^{-2}$, $\rho_{ISM} \simeq 0.03 M_\odot \text{pc}^{-3}$), we find $\gamma_{GMC} \simeq 12.8 \text{ Gyr}$. Note that the time-scale derived in G06 was derived for the evolution of the mass, not the energy, for reasons discussed above. We therefore use their mass-loss time-scale with the above parameters, and multiply it by $f = 3$ (as determined in Section 2.1) to get γ_{GMC} . This ensures that clusters lose the same amount of mass as in G06 with our definition of τ_{sh} for the evolution of E .

The increase in density and the decrease in mass as the result of GMC encounters leads to a reduction of the half-mass relaxation time-scale (τ_{rh} , see equation 9), hence GMC encounters eventually push collisionless clusters into the collisional regime. In the next section we discuss the effect of two-body relaxation.

3 TWO-BODY RELAXATION

3.1 Density evolution

To describe the effect of two-body relaxation on cluster evolution, we resort to the model of the evolution of an isolated globular cluster of Hénon (1965, hereafter H65). Most clusters are confined by a tidal field, but the model of the isolated cluster describes the early evolution of clusters that are initially dense compared to their tidal density (Gieles, Heggie & Zhao 2011), an assumption we adopt here.

The isolated cluster expands as the result of two-body relaxation with a central energy source, without losing mass. Although Hénon's model is highly idealized, more realistic models that include a stellar mass spectrum, the mass loss of stars (Gieles et al. 2010) and stellar-mass black holes (Breen & Heggie 2013), follow similar evolutionary tracks. Isolated clusters do lose some stars (Baumgardt et al. 2002), but we proceed with the simplifying assumption that the mass remains constant. The evolution of $\rho_h(E)$ is then simply (equation 1)

$$\frac{d\rho_h}{\rho_h} = 3 \frac{dE_{rlx}}{E}. \quad (7)$$

We note that this result is equivalent to that for tidal shocks for $f = 0$ (equation 3, i.e. shocks that only change the energy, not the mass). Shocks affect M more than E , such that ρ_h increases, while ρ_h decreases for two-body relaxation (equation 7).

3.2 time-scale

In a relaxation dominated system with a central energy source, the fractional energy change per τ_{rh} is approximately constant (Hénon 1961; H65)

$$\frac{dE_{rlx}}{E} = -\zeta \frac{dt}{\tau_{rh}}, \quad (8)$$

where $\zeta \simeq 0.08 - 0.10$ for equal-mass clusters (Hénon 1961; H65; Gieles et al. 2011; Alexander & Gieles 2012). For a constant Coulomb logarithm of $\ln \Lambda = 10$ and a constant stellar mean mass of $0.5 M_\odot$, τ_{rh} depends on M and ρ_h as

$$\tau_{\text{rh}} = \kappa \frac{M}{10^4 M_\odot} \left(\frac{\rho_h}{10^2 M_\odot \text{pc}^{-3}} \right)^{-1/2}. \quad (9)$$

For equal-mass systems, $\kappa \simeq 142 \text{ Myr}$ (Spitzer & Hart 1971). A stellar mass spectrum speeds the relaxation process up by about a factor of 2 for a globular cluster-like mass function (e.g. Kim et al. 1998). Young clusters contain more massive stars, causing the two-body relaxation process to be faster by a factor of 3 (at $\sim 100 \text{ Myr}$) to 20 (at $\sim 10 \text{ Myr}$) than in old globular clusters (Gieles et al. 2010). This effect could be included by making κ time dependent, but for consistency with other works we adopt the value of κ for equal-mass systems and we adopt a larger $\zeta = 0.5$.

In a collisional system that undergoes tidal shocks, the expansion by relaxation competes with the shrinking due to shocks and an equilibrium can be found by considering the respective time-scales of evolution. This is what we discuss in the next section.

4 COMBINED EFFECT OF SHOCKS AND RELAXATION

4.1 Evolution of the density: an equilibrium MRR

The density of a cluster evolving under the influence of tidal shocks and two-body relaxation can be found by adding the change in E due to tidal shocks (equation 4) and relaxation (equation 8)

$$\frac{dE}{E} = \frac{dE_{\text{sh}}}{E} + \frac{dE_{\text{rlx}}}{E}. \quad (10)$$

We use this in the general expression for the energy (equation 1) and express $dE_{\text{rlx}} = (\zeta \tau_{\text{sh}} / \tau_{\text{rh}}) dE_{\text{sh}}$ (equations 4 and 8). Combined with the mass evolution due to shocks (equation 2) we find

$$\frac{d\rho_h}{\rho_h} = \left(\frac{3}{f} - 5 + \frac{3}{f} \frac{\zeta \tau_{\text{sh}}}{\tau_{\text{rh}}} \right) \frac{dM}{M}. \quad (11)$$

This relation is equivalent to what we found earlier for shocks (equations 2 and 3), with the additional contribution of relaxation.

From equation (11) we see that for clusters with a low ratio $\tau_{\text{sh}}/\tau_{\text{rh}}$ the density increases quickly when M reduces, while for clusters with a high ratio $\tau_{\text{sh}}/\tau_{\text{rh}}$ the density decreases when M decreases. Because $\tau_{\text{sh}}/\tau_{\text{rh}} \propto \rho_h^{3/2}/M$, another way of describing this behaviour is that mass loss (due to shocks) dominates the evolution of low-density, massive clusters, while expansion (due to relaxation) dominates the evolution of dense, low-mass clusters.

The differential equation can be solved via a variable substitution $\rho_h/M^{2/3}$ and the solution is

$$\rho_h = \left[\frac{AM}{1 + (AM_i/\rho_{h,i}^{3/2} - 1)(M/M_i)^{1/2 - 9/27}} \right]^{2/3}, \quad (12)$$

where M_i and $\rho_{h,i}$ are the initial M and ρ_h , respectively, and $A = 0.1 M_\odot^{1/2} \text{pc}^{-9/2} (17f/9 - 1) \kappa / (\zeta \gamma_{\text{GMC}})$. This solution is only valid for $f > 9/17 \simeq 0.53$, which includes the value $f = 3$ that we derived in Section 2.1.

The differential equation given by equation (11) has an attractor solution with $\tau_{\text{sh}}/\tau_{\text{rh}} = \text{constant}$ ², i.e. then $\rho_h \propto M^{2/3}$, and

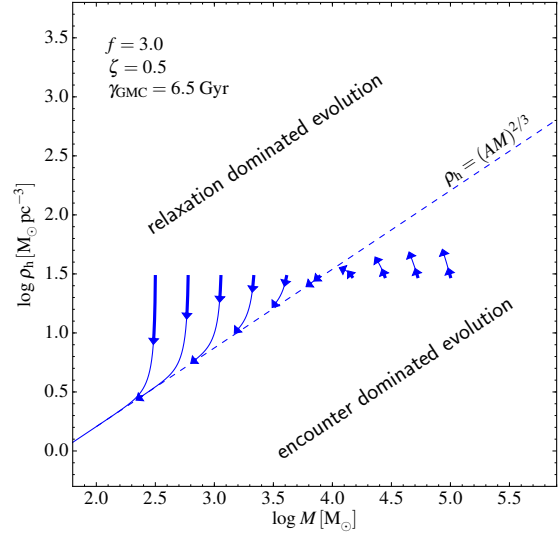


Figure 3. Evolution of ρ_h for different initial masses M_i , all with the same initial density of $\rho_{h,i} = 30 M_\odot \text{pc}^{-3}$. The first arrow along the tracks indicates an age of 30 Myr, and the second arrow marks 300 Myr. For this $\rho_{h,i}$, clusters less massive than $\log M_i/M_\odot \simeq 4$ are initially denser than the equilibrium relation $\rho_h = (AM)^{2/3}$ and expand by two-body relaxation towards it. More massive clusters first contract until they reach the same relation.

$r_h \propto M^{1/9}$. Filling in the parameters for the Milky Way (see Section 2.1), with $f = 3$ and $\zeta = 0.5$, the equilibrium MRR is

$$r_h \simeq 3.8 \text{ pc} \left(\frac{\gamma_{\text{GMC}}}{12.8 \text{ Gyr}} \right)^{2/9} \left(\frac{M}{10^4 M_\odot} \right)^{1/9}, \quad (13)$$

very similar to what is found for young extragalactic cluster populations in spiral galaxies: $r_h \simeq 3.75 \text{ pc} (M/10^4 M_\odot)^{0.1}$ (Larsen 2004, where we assumed $r_h = (4/3)r_{\text{eff}}$ to correct for projection). These clusters have ages ($\lesssim 100 \text{ Myr}$) comparable to τ_{rh} and τ_{sh} , suggesting that for these clusters, both two-body relaxation and GMC encounters are important. G06 and Elmegreen (2010) use the observed MRR and density dependent τ_{sh} to argue that cluster lifetimes depend on mass. Here we show that such an MRR is in fact the result of GMC encounters and two-body relaxation combined for clusters with $\tau_{\text{rh}} \simeq \tau_{\text{sh}} \lesssim \text{Age}$.

In Fig. 3 we show the evolution of $\rho_h(M)$ for different values of M_i . For all clusters we used $\rho_{h,i} = 30 M_\odot \text{pc}^{-3}$ and with the parameters chosen, the constant $A \simeq 0.02 M_\odot^{1/2} \text{pc}^{-9/2}$. The time evolution was found numerically. From equation (12) we see that for this $\rho_{h,i}$ clusters with $M_i \simeq \rho_{h,i}^{3/2}/A \simeq 10^4 M_\odot$ form on the equilibrium mass-density relation. For MRRs steeper than the equilibrium relation, low-mass clusters form in the relaxation dominated regime of the diagram and expand towards the equilibrium relation, while more massive clusters form in the shock dominated regime and contract initially.

the cluster shrinks as the result of stellar ejections, and expands as the result of GMC encounters, such that the equilibrium radius is an unstable equilibrium and clusters tend to move away from it. Our solution is an attractor and clusters will always move towards it.

² King (1958) points out that an equilibrium radius must exist when cloud interactions and relaxation are both at work. However, King assumed that

4.2 Dependence of the time-scales on gas properties

From the equilibrium MRR we can also derive the relation between τ_{sh} and τ_{rh} and their individual scaling relations

$$\tau_{\text{rh}} \simeq \frac{\zeta}{17f/9 - 1} \tau_{\text{sh}} \quad (14)$$

$$\tau_{\text{rh}} \simeq 302 \text{ Myr} \left(\frac{\gamma_{\text{GMC}}}{12.8 \text{ Gyr}} \right)^{1/3} \left(\frac{M}{10^4 M_{\odot}} \right)^{2/3} \quad (15)$$

$$\tau_{\text{sh}} \simeq 2822 \text{ Myr} \left(\frac{\gamma_{\text{GMC}}}{12.8 \text{ Gyr}} \right)^{1/3} \left(\frac{M}{10^4 M_{\odot}} \right)^{2/3} \quad (16)$$

$$\tau_{\text{dis}} \simeq 940 \text{ Myr} \left(\frac{\gamma_{\text{GMC}}}{12.8 \text{ Gyr}} \right)^{1/3} \left(\frac{M}{10^4 M_{\odot}} \right)^{2/3}. \quad (17)$$

Here $\tau_{\text{dis}} = \tau_{\text{sh}}/f$ is the time-scale for the evolution of M . The dependence of all time-scales on $\gamma_{\text{GMC}}^{1/3}$ shows that variations in the properties of molecular gas only mildly affect the evolution. This is because for lower γ_{GMC} , the clusters are denser and lose less mass than what is expected from the linear dependence of τ_{sh} on γ_{GMC} in equation (4). The constants of proportionality in equations (13) and (17) depend mildly on the adopted value for f , for $f = [1, 3, 10]$ they are $[4.3, 3.8, 3.7]$ pc and $[648, 941, 1032]$ Myr, respectively. We note that only for $f < 9/17$ the scaling between $\rho_{\text{h}}(M)$ becomes f -dependent, because then $\rho_{\text{h}} \propto M^{\frac{3}{f}-5}$. This excludes the evolution at a constant density ($f = 3/5$), but could allow for evolution at constant r_{h} ($f = 1/2$).

We note that on the equilibrium MRR the dimensionless mass-loss rate $\xi \equiv -\dot{M}\tau_{\text{rh}}/M = \tau_{\text{rh}}/\tau_{\text{dis}} \simeq 0.3$ is higher than that of clusters in isolation ($\xi \simeq 0.01$, e.g. Aarseth & Heggie 1998) or in a static galactic tidal field ($\xi \simeq 0.05$, e.g. Hénon 1961).

5 CONCLUSIONS

In this work we show that the interplay between two-body relaxation and tidal shocks leads to an MRR for star clusters in which the radii are almost independent of their masses ($r_{\text{h}} \propto M^{1/9}$). Based on the ISM properties in the solar neighbourhood we estimate that these processes together could be responsible for the typical cluster radius of ~ 3 pc for low-mass ($M \lesssim 10^5 M_{\odot}$), young (\lesssim few 100 Myr) star clusters.

The mild dependence of r_{h} on M implies that the time-scale for disruption by GMC encounters depends on M as $\tau_{\text{dis}} \propto M^{2/3}$ (equation 17). This mass dependence is similar to what is found for τ_{dis} as the result of evaporation in a tidal field ($\tau_{\text{dis}} \propto M^{3/4}$, for a constant Coulomb logarithm and a constant stellar mass, Baumgardt 2001) and what was derived empirically by Lamers et al. (2005). This means that GMC encounters and relaxation contribute in a similar way to ‘turning over’ a power-law GCMF, as the longer term evaporation process in a galactic tidal field.

Elmegreen (2010) uses the high surface densities of molecular gas in $z \sim 2-3$ galaxies (e.g. Tacconi et al. 2010) to argue that τ_{dis} due to cloud encounters is short enough to turn over the GCMF in the early evolution of globular clusters. This would imply that all globular clusters could have formed with a similar power-law mass function as YMCs in the nearby Universe (power-law with index -2 , see e.g. Zhang & Fall 1999, for the case of the clusters in the Antennae galaxies). Elmegreen (2010) estimates that the value of γ_{GMC} in equation (5) needs to be a factor of ~ 16 smaller than in the solar neighbourhood for this to work in about 500 Myr. Here we show that accounting for internal evolution of clusters during this disruption phase requires γ_{GMC} to be a factor of $16^3 = 4096$ lower

instead. Although the observed properties of some high-redshift galaxies may well be consistent with such short values of τ_{dis} , we note these galaxies are not Milky Way progenitors because they are more massive than the Milky Way today.

Cluster mass loss as the result of two-body relaxation in the Galactic tidal field is not sufficient to explain the absence of low-mass GCs in the outer halo (Baumgardt 1998; Vesperini 2001). Elmegreen (2010) proposes that the additional mass loss due to tidal shocks with passing GMCs can alleviate this ‘GCMF problem’. We demonstrate that τ_{dis} due to GMC encounters has indeed the correct mass dependence, but we also show that due to the self-limiting nature of tidal shocks, the required ISM properties for this to work are more extreme. In a follow-up study we consider this in more detail, aided by results from hydrodynamical simulations of Milky Way formation in the cosmological context.

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REFERENCES

- Aarseth S. J., Heggie D. C., 1998, MNRAS, 297, 794
- Alexander P. E. R., Gieles M., 2012, MNRAS, 422, 3415
- Baumgardt H., 1998, A&A, 330, 480
- Baumgardt H., 2001, MNRAS, 325, 1323
- Baumgardt H., Hut P., Heggie D. C., 2002, MNRAS, 336, 1069
- Binney J., Tremaine S., 1987, Galactic dynamics. Princeton Univ. Press, Princeton, NJ
- Breen P. G., Heggie D. C., 2013, MNRAS, 436, 584
- Elmegreen B. G., 2010, ApJ, 712, L184
- Elmegreen B. G., Hunter D. A., 2010, ApJ, 712, 604
- Elson R. A. W., Fall S. M., Freeman K. C., 1987, ApJ, 323, 54
- Fall S. M., Chandar R., Whitmore B. C., 2009, ApJ, 704, 453
- Gieles M., Baumgardt H., Heggie D. C., Lamers H. J. G. L. M., 2010, MNRAS, 408, L16
- Gieles M., Heggie D. C., Zhao H., 2011, MNRAS, 413, 2509
- Gieles M., Portegies Zwart S. F., Baumgardt H., Athanassoula E., Lamers H. J. G. L. M., Sipior M., Leenaarts J., 2006, MNRAS, 371, 793 (G06)
- Gnedin O. Y., Lee H. M., Ostriker J. P., 1999, ApJ, 522, 935
- Gnedin O. Y., Ostriker J. P., 1999, ApJ, 513, 626
- Hénon M., 1959, Annales d’Astrophysique, 22, 126
- Hénon M., 1960, Annales d’Astrophysique, 23, 474
- Hénon M., 1961, Annales d’Astrophysique, 24, 369; translation: ArXiv:1103.3499
- Hénon M., 1965, Annales d’Astrophysique, 28, 62; translation: ArXiv:1103.3498 (H65)
- Jaffe W., 1983, MNRAS, 202, 995
- Kharchenko N. V., Piskunov A. E., Röser S., Schilbach E., Scholz R.-D., 2005, A&A, 438, 1163
- Kim S. S., Lee H. M., Goodman J., 1998, ApJ, 495, 786
- King I., 1958, AJ, 63, 465
- King I. R., 1966, AJ, 71, 64
- Kruijssen J. M. D., 2015, MNRAS, 454, 1658
- Kundic T., Ostriker J. P., 1995, ApJ, 438, 702
- Lamers H. J. G. L. M., Gieles M., Bastian N., Baumgardt H., Kharchenko N. V., Portegies Zwart S., 2005, A&A, 441, 117
- Larsen S. S., 2004, A&A, 416, 537
- Larson R. B., 1981, MNRAS, 194, 809
- Mackey A. D., Gilmore G. F., 2003, MNRAS, 338, 85

- Ostriker J. P., Spitzer L. J., Chevalier R. A., 1972, *ApJ*, 176, L51
- Plummer H. C., 1911, *MNRAS*, 71, 460
- Portegies Zwart S. F., McMillan S. L. W., Gieles M., 2010, *ARA&A*, 48, 431
- Scheepmaker R. A., Haas M. R., Gieles M., Bastian N., Larsen S. S., Lamers H. J. G. L. M., 2007, *A&A*, 469, 925
- Spitzer L., 1987, *Dynamical evolution of globular clusters*. Princeton, NJ, Princeton University Press, 1987, 191 p.
- Spitzer L. J., 1958, *ApJ*, 127, 17
- Spitzer L. J., Hart M. H., 1971, *ApJ*, 164, 399
- Tacconi L. J. et al. 2010, *Nature*, 463, 781
- Terlevich E., 1987, *MNRAS*, 224, 193
- Urquhart J. S. et al. 2014, *MNRAS*, 443, 1555
- van den Bergh S., 2006, *AJ*, 131, 1559
- Vesperini E., 2001, *MNRAS*, 322, 247
- Weinberg M. D., 1994, *AJ*, 108, 1398
- Wielen R., 1985, in Goodman J., Hut P., eds, *Proc. IAU Symp. 113, Dynamics of Star Clusters Dynamics of open star clusters*. Reidel, Dordrecht, p 449
- Zepf S. E., Ashman K. M., English J., Freeman K. C., Sharples R. M., 1999, *AJ*, 118, 752
- Zhang Q., Fall S. M., 1999, *ApJ*, 527, L81

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